



# CSA Pricing

Roland Lichters

QuantLib User Meeting, 30 November - 1 December 2015

# Agenda

About us

Introduction

CSA Pricing

# Agenda

About us

Introduction

CSA Pricing

## About us

www.quaternion.com

### News:

- ▶ Increasing client base in Europe
- ▶ Added Quaternion Risk Management US Inc.
- ▶ 25 staff and growing in US, IE, UK and DE
- ▶ Hybrid offering: Software solutions rooted in quant consulting services for Tier 1 banks

# Agenda

About us

Introduction

CSA Pricing

# Introduction

Exciting things to do in quant finance  
... and using QuantLib

# Introduction

Tenor and cross currency basis:  
Multi curve pricing, OIS discounting

# Introduction

Negative rates:

Review and revise your pricing models



# Introduction

## Derivatives catch up with loans: Valuation Adjustments for Credit, Funding, Capital

# Introduction

Margin requirements for centrally cleared and  
OTC derivatives:

More VAs - Margin Value Adjustment

# Introduction

Tighter supervision of Internal Model banks:  
Credit Exposure simulation for derivatives

# Introduction

New standard approach for credit risk capital  
(SA-CCR):

Impact analysis, comparison to CEM Add-On,  
and internal model EAD

# Introduction

Optimization of capital and funding cost:

A combination of all of the above

# Introduction

Stress tests and sensitivity analysis on top of  
xVA and exposures

⇒ It is has got more complicated and more  
computationally demanding

# Introduction

So there is a need for efficient, clever and transparent tools to cover all this.

# Agenda

About us

Introduction

CSA Pricing



# CSA Pricing and yet another VA

## Ideal CSA

- ▶ Symmetric
- ▶ Cash collateral in single currency
- ▶ Daily margining

## CSA Pricing and yet another VA

Real CSAs are often more complex

- ▶ One-sided thresholds
- ▶ Optional bond collateral
- ▶ Rating triggers
- ▶ Collateral currency choice
- ▶ Cash collateral compounding rate (Eonia) shifted and floored at zero

# CSA Pricing and yet another VA

New regulations: Increased capital charges for residual risks e.g. due to asymmetric CSAs

Increased cost of risk mitigation measures

Trend to simplify CSAs (ISDA)

## CSA Pricing and yet another VA

### A real case:

- ▶ Harmonize CSAs for adequate compensation, i.e. price the features
- ▶ Portfolio: A few thousand Swaps, FX Swaps, Bermudan Swaptions, Inflation Swaps, BMA Swaps, CDS, and a bunch of structured products

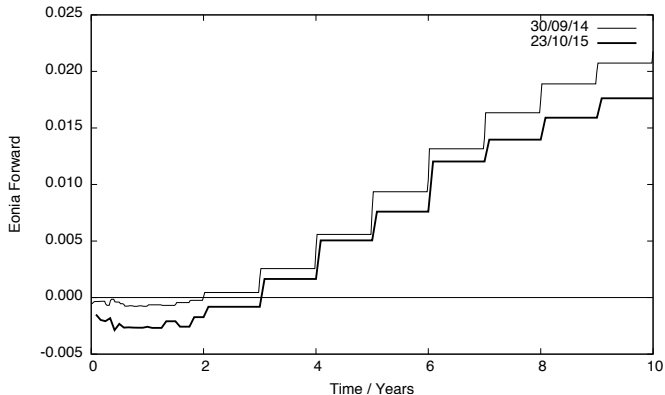
# What does this involve?

## At the high level

- ▶ One-sided thresholds  
⇒ **CVA, DVA, FVA**
- ▶ Bond collateral and rating triggers  
⇒ **Credit modelling**
- ▶ Collateral currency choice  
⇒ **Cross currency basis modelling**
- ▶ Cash collateral compounding rate (Eonia)  
shifted and floored at zero ⇒ **See next slides**

# Market

## Eonia curves as of 30/09/2014 and 23/10/2015



# Methodology

The Eonia floor feature in a CSA has two effects:

1. It affects the fair value of each derivative in the netting set
2. It affects the fair value of future interest cash flows paid/received in the margining process

# Discounting

## 'Ordinary' OIS Discounting

$$\text{Discount}(T) = \mathbb{E} \left[ e^{-\int_0^T r(s) ds} \right]$$

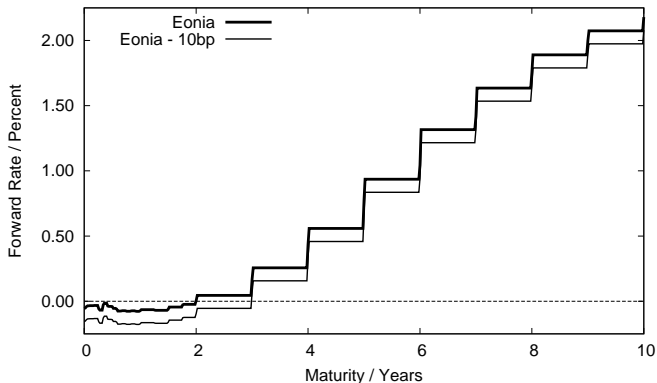
## 'Floored' OIS Discounting

$$\text{Discount}(T) = \mathbb{E} \left[ e^{-\int_0^T r^+(s) ds} \right]$$

does not have closed form, but approximate solutions.

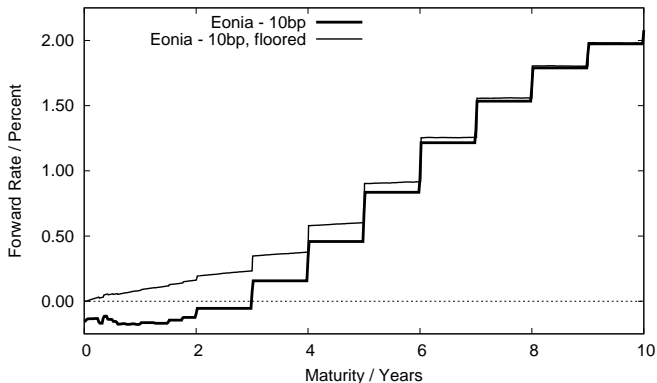


# Discounting



**Figure:** EONIA forward curve as of 30/09/2014 with negative rates up to 2 years. Under the CSA collateral is paid in EUR and based on EONIA - 10 bp.

# Discounting



**Figure:** Shifted EONIA forward curve compared to the forward curve with collateral floor; Hull-White parameters are  $\lambda = 0.05$  and  $\sigma = 0.004$ .

# Swap Pricing

Needs more than discounting, floating leg:

$$\Pi_{Float} = \mathbb{E} \left[ \sum_{i=1}^n L(t_{i-1}, t_i) \times \delta(t_{i-1}, t_i) \times D(t_i) \right]$$

$$L(t_{i-1}, t_i) = \frac{1}{\delta(t_{i-1}, t_i)} \left( e^{\int_{t_{i-1}}^{t_i} r(s) ds} - 1 \right)$$

$$D(t_i) = e^{-\int_0^{t_i} r^+(s) ds}$$

## Swap Pricing

It is tempting to build the 'floored' discount curve, keep the forward curves unchanged and to do curve pricing as usual.

Unfortunately, this does not yield the 'exact' Swap value.

Only full MC pricing yields the 'exact' price, even for a vanilla Swap.

# Swap Pricing

The error is noticeable but quite small.

| Term | no floor | with floor | diff  | approx.  | error  |
|------|----------|------------|-------|----------|--------|
| 2    | -665.21  | -663.02    | 2.19  | -663.26  | -0.24  |
| 3    | -1135.64 | -1129.83   | 5.81  | -1130.54 | -0.71  |
| 4    | -1459.73 | -1450.40   | 9.34  | -1451.84 | -1.44  |
| 5    | -1743.69 | -1730.53   | 13.16 | -1732.88 | -2.35  |
| 7    | -2216.14 | -2195.89   | 20.24 | -2200.47 | -4.58  |
| 10   | -2757.42 | -2728.46   | 28.96 | -2735.91 | -7.45  |
| 12   | -3056.87 | -3022.23   | 34.64 | -3031.87 | -9.64  |
| 15   | -3485.65 | -3442.96   | 42.69 | -3455.76 | -12.80 |
| 20   | -4204.30 | -4148.54   | 55.77 | -4165.78 | -17.24 |

**Table:** Vanilla swaps, 4% fixed vs Euribor 6m flat, Hull White model with  $\lambda = 0.01$  and  $\sigma = 0.005$ , market data as of 30/06/2015. Approx: Only discount curve replaced, forward curve unchanged

# Margin Effect, Eonia Floor Value

Move on to the second effect:  
Impact on interest on collateral

## Margin Effect, Eonia Floor Value

Without Eonia floor, the value of collateral interest cash flows is

$$\Pi_{NotFloored} = \mathbb{E} \left[ \sum_i C(t_i) \cdot r(t_i) \cdot \delta_i \cdot D(t_{i+1}) \right]$$

with

- ▶  $C(t_i)$ : posted collateral
- ▶  $r(t_i)$ : overnight rate applicable to period  $(t_i, t_{i+1})$
- ▶  $\delta_i$ : day count fraction for period  $(t_i, t_{i+1})$
- ▶  $D(t_{i+1})$ : stochastic discount factor

## Margin Effect, Eonia Floor Value

With floored Eonia rates, the value of collateral interest cash flows is

$$\Pi_{Floored} = \mathbb{E} \left[ \sum_i \tilde{C}(t_i) \cdot r^+(t_i) \cdot \delta_i \cdot \tilde{D}(t_{i+1}) \right]$$

where  $\tilde{C}$  and  $\tilde{D}$  denote floor-induced modified collateral amounts and stochastic discount factors.



# Margin Effect, Eonia Floor Value, COLVA

In summary

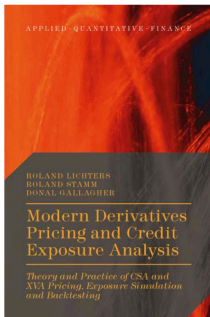
$$\Pi_{Floor} = \Pi_{Floored} - \Pi_{NotFloored}$$

$$= \mathbb{E} \left[ \sum_i \left( \tilde{C}(t_i) \tilde{D}(t_{i+1}) \delta_i (r(t_i))^+ - C(t_i) D(t_{i+1}) \delta_i r(t_i) \right) \right]$$

$$\approx \mathbb{E} \left[ \sum_i C(t_i) D(t_{i+1}) \delta_i (-r(t_i))^+ \right].$$

So-called **COLVA**, see Burgard-Kjaer, or ...

# Methodology



## Modern Derivatives Pricing and Credit Exposure Analysis

Theory and Practice of CSA and XVA Pricing, Exposure Simulation and Backtesting

Roland Lichters, Roland Stamm, Donal Gallagher

Hardcover 9781137494832 £60.00 / \$95.00

palgrave  
macmillan

Available from all good booksellers or online at [www.palgrave.com](http://www.palgrave.com)

To order in the USA or Canada: T: 888-330-8477  
If you are in Australia or New Zealand: E: [palgrave@macmillan.com.au](mailto:palgrave@macmillan.com.au)  
To order in UK or rest of world: T: +44 (0)1256 302866, E: [orders@palgrave.com](mailto:orders@palgrave.com)



## Margin Effect, Eonia Floor Value, COLVA

$$\Pi_{Floor} \approx \mathbb{E} \left[ \sum_i C(t_i) D(t_{i+1}) \delta_i (-r(t_i))^+ \right]$$

is the price of a floor

- ▶ paying off when overnight rates are negative, i.e. currently in the money
- ▶ with stochastic notional given by the amount of posted collateral
- ▶ potentially with significant correlation between notional and rate, depending on netting set composition

# Implementation

Quantify floor effects by means of

1. Floor-induced single-trade pricing for interest rate, FX and inflation derivatives using bespoke pricing engines
2. Monte Carlo simulation of the netting set collateral in conjunction with simulation of the compounding rate

QuantLib applied in Quaternion Risk Engine

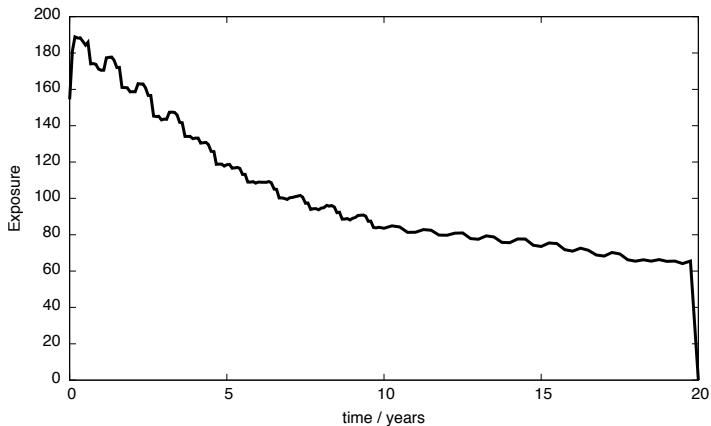
# Implementation

## Monte Carlo Simulation Framework

- ▶ IR: Linear Gauss Markov models
- ▶ FX: Geometric Brownian Motion driven by stoch. IR differential
- ▶ INF: Jarrow Yildirim
- ▶ CR: Cox Ingersoll Ross and Black Karasinski

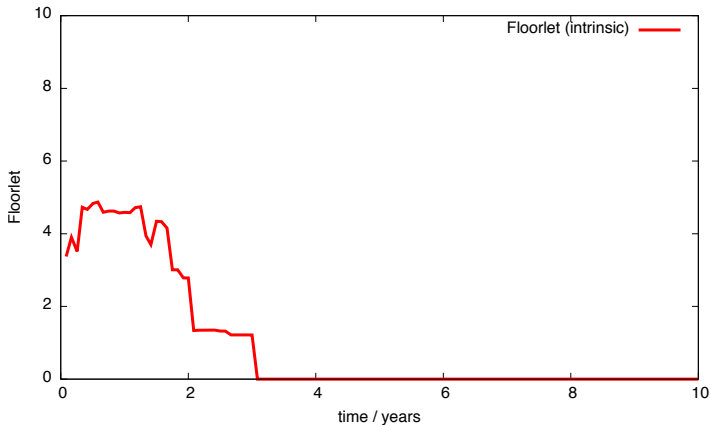
# Example Portfolio as of 23/10/2015

## Exposure evolution



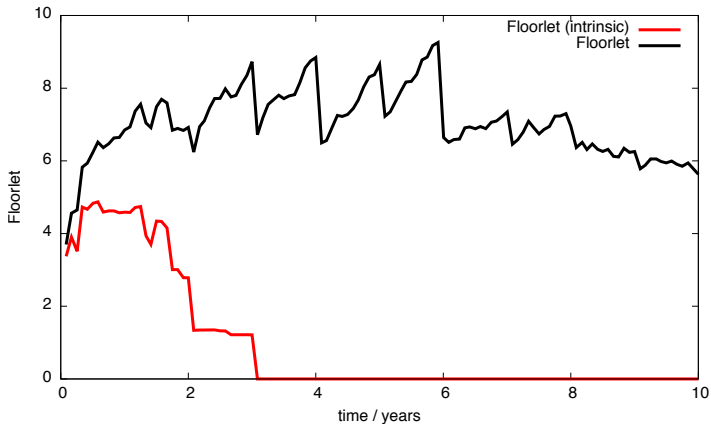
# Example Portfolio

## Floorlets, intrinsic values



# Example Portfolio

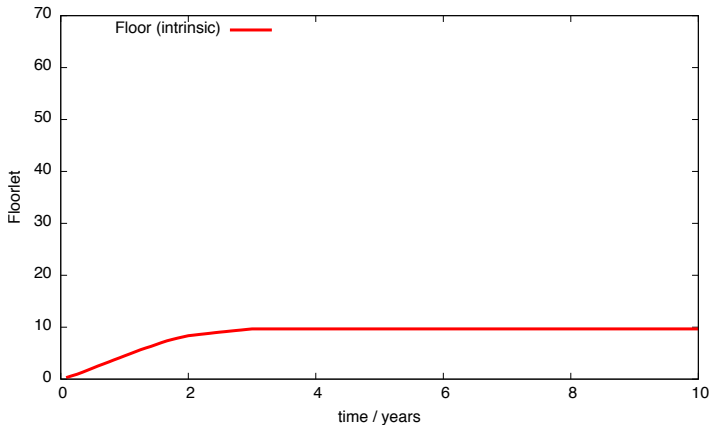
## Floorlets





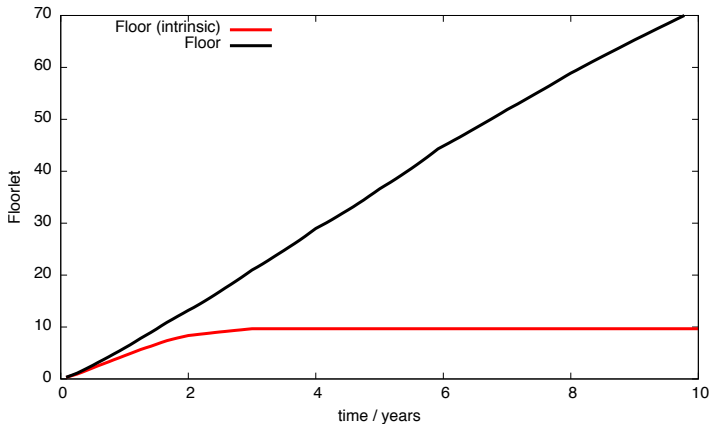
# Example Portfolio

## Floor, intrinsic



# Example Portfolio

## Floor



# Summary

## CSA 'Eonia Floor' value

- ▶ can be seen as COLVA
- ▶ should be computed by full MC simulation due to the correlation between posted collateral and compounding rate
- ▶ has significant time value
- ▶ is exposed to significant model risk (rate distribution for negative rates)

# Questions