Market Models vs. Replication Strategies in incomplete Commodity Markets

M. Dietrich, P. Heider
Uniper Global Commodities

QuantLib User Meeting 2016
Düsseldorf
08-12-2016

Introduction

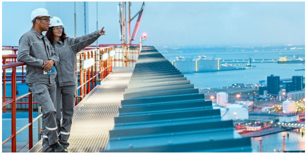
Asset Backed Trading

Goal: secure the flexibility of physical assets ...

Challenges in Incomplete (Commodity) Markets

- large bid/offer spreads (illiquidity)
- option markets are not as developed as FX, equity, .. markets
- typically used to hedge real options (assets), i.e. non tradable flexibility classes
- mathematical foundations are questionable, e.g. power is not storable





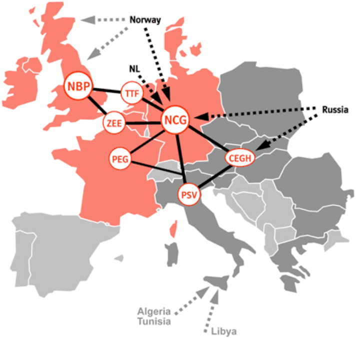
The European Gas Market

Major trading *hubs* in Europe:

- the National Balancing Point (**NBP**) in UK
- Title Transfer Facility (**TTF**) in the Netherlands
- **NCG** and **GASPOOL** in Germany
- **PEGn**, **PEGs** in France

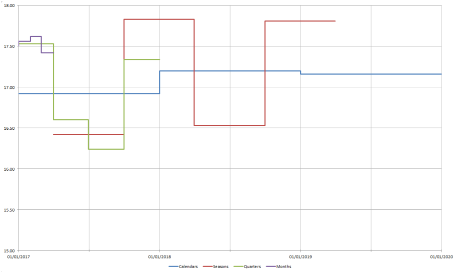
Hubs are connected:

- UK market and continental Europe are connected by the **interconnector**
- Major pipeline system in continental Europe and links to upstream providers



Gas Futures Market

- monthly, quarterly, seasonally, yearly contracts
- seasonal contracts are *summer* (Apr-Sep) and *winter* (Oct-Mar)
- **cascading** of fwd contracts: on expiry these futures are replaced with equivalent futures with shorter delivery periods
- day-ahead forwards, weekend ahead, ...



Forward Curves

- Forward products of different granularity form an overlapping term structure.
- *Shaping* models are used to derive an arbitrage-free (artificial) monthly, daily or hourly granular term structure to value flexibility.

Gas Storage

Storages ensure security of supply and stabiliy of prices:

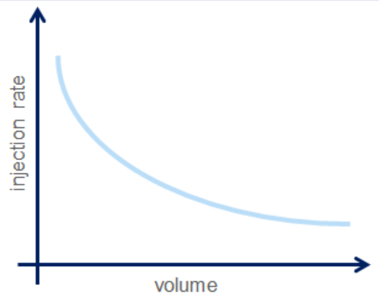
- Seasonal demand for gas is linked to gas heating of houses
- Operation of gas-fired power plants are often designed for peak delivery
- decouples upstream issues from downstream supply

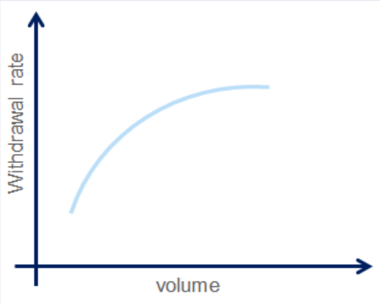
Gas Storage or Hydro Pump Station

Physical Constraints

Underground storages are e.g. depleted gas/oilfield or salt caverns

- *Bergrechtliche* constraints
- costs might depend on time, volume level, temperature etc.
- there might be time dependent minimum and maximum fill level requirements
- injection/withdrawal rates are volume dependent





Gas Storage - Managing Supply and Flexibility

Gas Storage Optionality

- . . . on a seasonal spread with price of gas as the underlying of the option
- . . . to choose between injection to withdrawal or do nothing
- . . . to adjust the timing of hedges in the forward and option market
- . . . to deliver to multiple pipelines or hubs

Optionality is limited by operational constraints:

- rate of injections/withdrawals depend on level of gas in storage
- gas may have to be injected/withdrawed due to contractual obligations

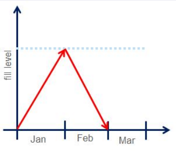
Intrinsic Storage Valuation

Example Storage

- injection/withdrawal: 1 MWh / day
- delivery: 1st of Jan till 31st of March
- start / end fill level 0 MW, no costs
- prices as of 8th Dec (made up):
  Jan 17.56, Feb 17.62, Mar 17.42

Intrinsic as of 8th Dec

- Use forwards to lock in: buy gas in Jan, sell in Feb



You have *flexibility*:

- Forward prices can change, till end of Dec. You have choices, e.g.:
- buy Jan sell Feb, $(\text{Feb} - \text{Jan})^+$
- buy Jan sell Mar, $(\text{Mar} - \text{Jan})^+$
- buy Feb sell Mar, $(\text{Mar} - \text{Feb})^+$

$\rightarrow$ **value the storage as the maximum of all possible spread options:**

- This value is larger than the intrinsic value.
- The difference is called the *extrinsic* value.
- Valuation depends on forward vol and correlation.

Replication as Basket of (Time)-Spread Options (I)

Flexibility in Storages

- Consider storage as set of time spread options to swap volumes from one period to another.
- Replication Strategy: The time spread options and forwards have to be allocated in such a way, that at any time all physical constraints of the storage are satisfied.
- A lower bound of the storage value is the maximum value of all possible replication strategies.
  $\rightarrow$ *complex linear optimization problem, needs commercial solver*
- This approach is essentially model-free if relevant time spread options are traded in the markets; otherwise use standard models to price the options.

Replication as Basket of (Time)-Spread Options (II)

Advantages

- Intuitive decomposition in standard products
- Computationally very efficient, availability of all Greeks
- tailor-made for physical assets with many constraints

Questions to Solve

- Correlation structure to value time spread options (e.g. correlation smiles, liquidity adjustments, ...)
- Time spread option model for illiquid / incomplete markets
- Undervaluation of fast churn storages: no spot feature
- Delta-profiles are changing due to wobbeling maximizing allocation strategy

Swing Contracts

Swing

Swing contracts are path dependent options allowing the holder to exercise a certain right multiple times over a specified period of time but only one right per exercise.

Might be the right:

- to receive a fixed price forward
- to receive an indexed price forward
- to receive a forward out of a list of market areas (multi-location)

Volume Swing (typical for gas/power)

The volume swing involves further restrictions like upper and lower bounds for certain time periods.

Swing Contracts - Managing Flexibility

Swing contracts protect holder against excessive rises in energy prices and volume risks.

Example of different flexibility classes:

- 0/365: strip of daily outright options
- 180/180: exposed to forward and time spread prices
- 90/180: mixure of both cases above
- month-ahead: strike = spot minus average month ahead price

Replication as Basket of Forwards, Calls and Spread Options (I)

Decomposition

- Use forwards to build min take profile.
- Add call options to account for *up-swing* rights.
- Use time spread options to account for flexibility of allocation time.

Example Swing

- delivery: 1st of Jan till 31st of March
- daily take: 0 - 1 MWh
- min take: 30 MWh, max take: 60 MWh
- strike shall be 17.53
- prices as of 8th Dec (made up): Jan 17.56, Feb 17.62, Mar 17.42

intrinsic: $\rightarrow$ lock in Jan and Feb

You have *flexibility*:

- forward in Jan and Feb, plus spread option to swap Jan and Mar
- or, forward in Jan, plus call on Feb
- ...

$\rightarrow$ **value the swing as the maximum of all possible decompositions:**

- Valuation depends on forward vol and correlation

Valuation of Swing and Storage Flexibility

Valuation

- Valuation of the flexibility is required - premiums are required *upstream* and *downstream* (eg. sales contracts)
- Modeling the relevant physical constraints requires advanced optimization techniques

Hedging

- Provision of reliable Greeks for position management
- Aggregation of Greeks to tradeable products
- Efficient numerical algorithms are required

Simulation Based Valuation (I)

Toy Model - in QuantLib

- Use simulation model (eg spot and forward) to value structured deals
- For example, in Quantlib available spot model by Kluge et al.
  `class ExtOUWithJumpsProcess`

$$S_t = \exp\left(f(t) + X_t + Y_t\right)$$
$$dX_t = -\alpha X_t dt + \sigma dW_t, \qquad dY_t = -\beta Y_{t-} dt + J_t dN_t$$

- Note, limited use as too few risk factors involved.

Advantages of Simulation Based Valuation

- Models can be easily replaced, valuation and simulation can be separated.
- Complex deals can be valued by using flexible payoff scripting.
- Spot optimization of the path dependent optionality

Simulation Based Valuation (II)

Numerical Approaches

- Use LSMC approach for high dimensional market models.
- Use PDE (PIDE) whenever possible to get stable greeks and less algorithmic tunning parameters.
- Use PDE solution as benchmark to find optimal LSMC setup for highly complex termsheets.

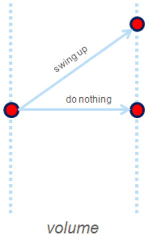
Challenges

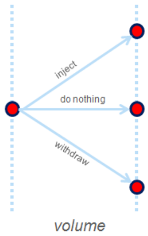
- Swing contract is a multi-callable Bermudan option ...
- Proper specification of algorithmic parameters (regression polynomials, number of simulations for calibration versus valuation, interpolation in admissible space, ...)
- High demand on computational resources (memory management, parallelization, automated Greek engine, ...)
- Standard LSMC algorithm needs to be enhanced for physical constraint types $\rightarrow$ requires *complicated dynamic programming* (or stochastic programming)

Dynamic Programming

Bellmann's Principle

$$F\left(t_n, \vec{X}_{t_n}, \vec{Y}_{t_n}\right) = \max\left(\Pi\left(t_n, \vec{X}_{t_n}, \vec{Y}_{t_n}\right), e^{-r(t_{n+1}-t_n)}\mathbb{E}^*_{t_n}\left(F\left(t_{n+1}, \vec{X}_{t_{n+1}}, \vec{Y}_{t_{n+1}}\right)\right)\right)$$

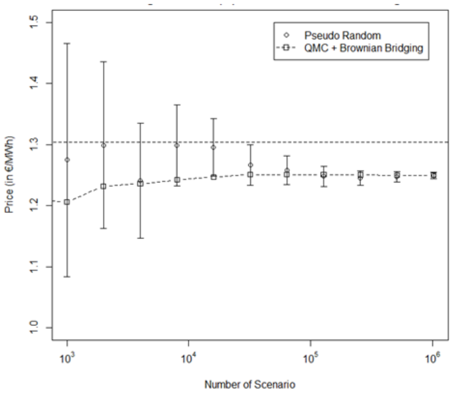


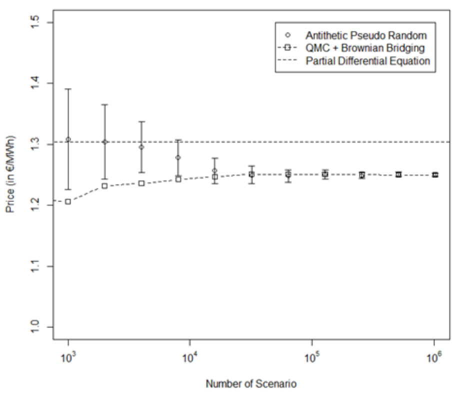


- Use linear programing to calcuate admissible physical states
- Interpolation in physical state space is necessary and interpolation methods is important
- Problem dimensionality depends on number of market risk factors and number of volume/time integral constraints

Convergence of LSMC

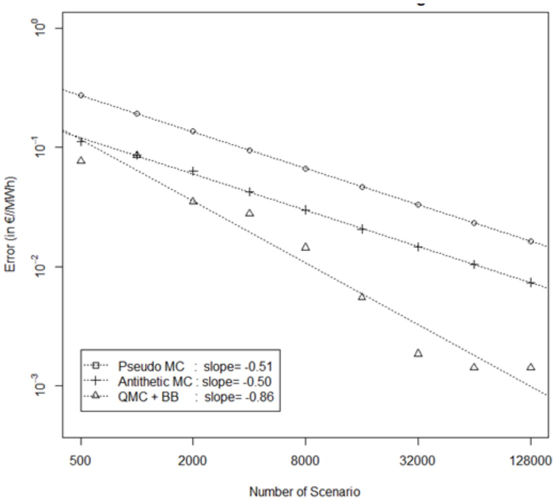
- Use standard OTC traded swing to compute benchmark value with P(I)DE using a very fine grid.





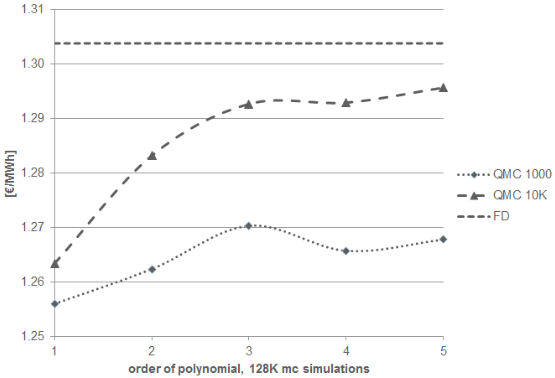
Order of Convergence

- We can use the benchmark to assess the convergence rate of different implementation flavours



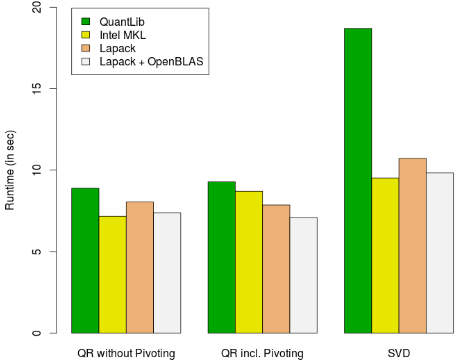
Regression Polynomials

- The benchmark allows to study the impact of the regression (polynomials) basis.



LSMC: Fast Least Squares Regression

Swing pricing via LSMC spends a decent amount of the CPU time on ordinary least squares regression.



Since the QR decomposition without column pivoting has numerical problems if A is rank-deficient, the singular value decomposition is often the method of choice for LSMC simulations.

QuantLib Building Blocks

Our pricing libary is mainly written in Java/Scala (JVM based). We utilise several numerical work horses from QuantLib via SWIG. Our own C++ extension to QuantLib is rather small.

Most relevant building blocks are

- Volatility Modelling
- Operator Splitting schemes for PDE solution
- Brownian Bridge and Quasi-Monte Carlo

Summary

Replication

- fast computation
- suitable for complicated physical constraints
- stable valuation
- breakdown into tradable products
- traders' intuition
- lower bound
- Δ-profiles can be shaky over time

Simulation

- captures the nature of the problem
- spot and forward dynamics
- flexibility in model choice
- too optimistic for incomplete markets
- LSMC: short time to market
- LSMC: high dimensional market models
- choose P(I)DE whenever possible

Disclaimer

The views, opinions, positions or strategies expressed in this presentation are those of the authors and do not necessarily represent the views, opinions, positions or strategies of and should not be attributed to Uniper Global Commodities.