Solvency II Regulation
How QuantLib can help

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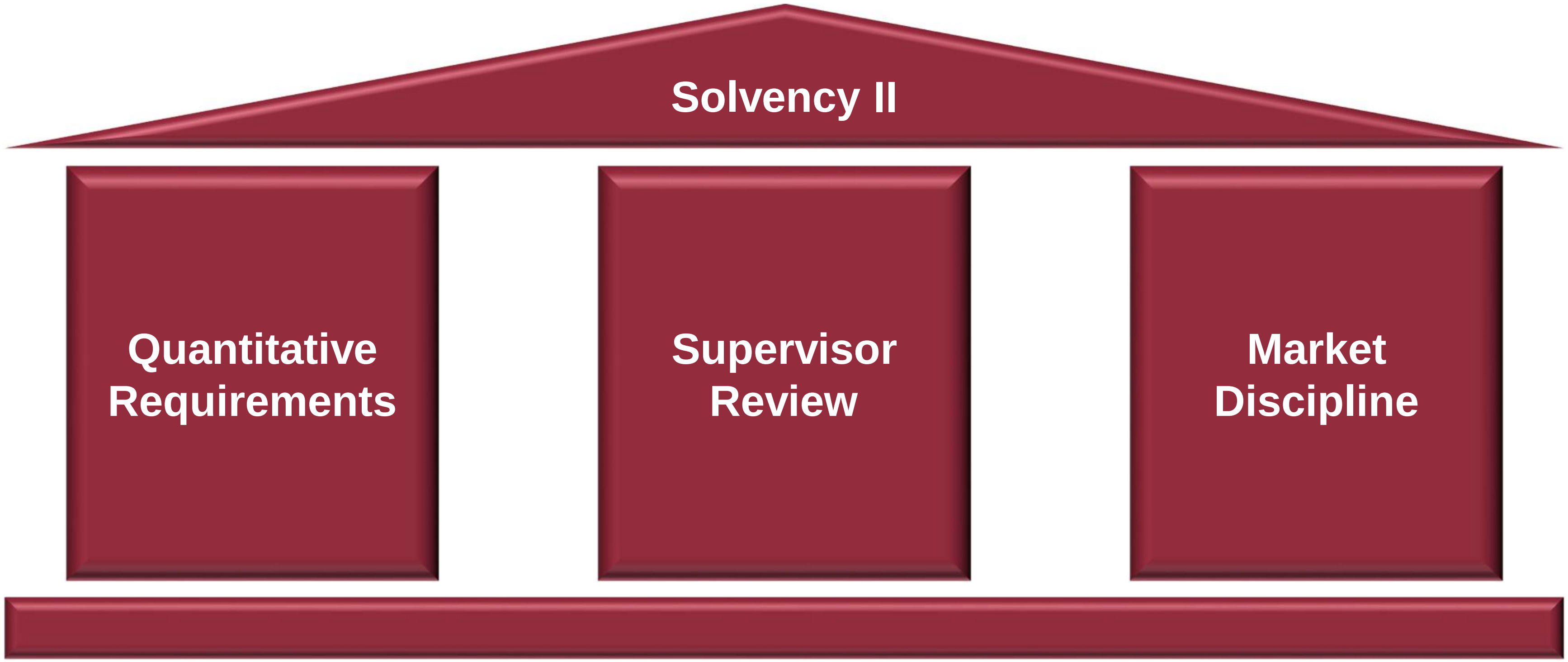
QuantLib User Meeting, Düsseldorf, 8.12.2016

Agenda

- *Solvency II and financial modelling*
- *Building economic scenario generator using QuantLib*
- *Interest rate modelling for Solvency II*



- In force since 1 January 2016
- Goal is to establish a single regulatory framework for EU insurers and reinsurers
- Inspired by Basel II / III but quite different in details
- Requires market consistent valuation of insurance liabilities

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Insurance Portfolio

Legal, Property & Casualty	Life & Pensions	Health Insurance
Focus on actuarial estimation of liability cash flows	Main challenge is the valuation of embedded financial options and guarantees	Combination of actuarial and financial approaches

Valuation approach depends on line of business

Example: *Unit linked pension plan (very simplified)*

- *At inception* $t = 0$*:* minimum guarantee rate g is fixed for pay-out phase
- *Accumulation:* customer contributions are invested in equity index (without guarantee)
- *At retirement* $t = T$*:* the customer savings $S(T)$ are reinvested in risk the free zero bond at interest rate $r(T)$
- *At maturity* $t = T + M$*:* customer becomes $S(T)\exp\big(M\max\big(g, r(T)\big)\big)$
- *Financial guarantees* in this example are equivalent to zero bond option with notional indexed by equity index (option maturity T bond maturity $T + M$).
- Value of financial options and guarantees depends on
  - *Equity volatility*
  - *Rates volatility*
  - *Correlation*

How exotic insurance contracts can be

Conventional life and pension insurance policies are much more complicated

- Pay-out of conventional life and pension insurance depends on the performance of investment portfolio.
- Usually a minimum performance rate is guaranteed.
- Some health insurance policies are exposed to inflation risk.
- Value of financial options and guarantees in general depends on volatility and correlations in
  - *Interest rates*
  - *Equity and property indices*
  - *Credit spreads*
  - *Inflation*
  - *FX*

Valuation of Insurance Policies by Monte-Carlo Simulation

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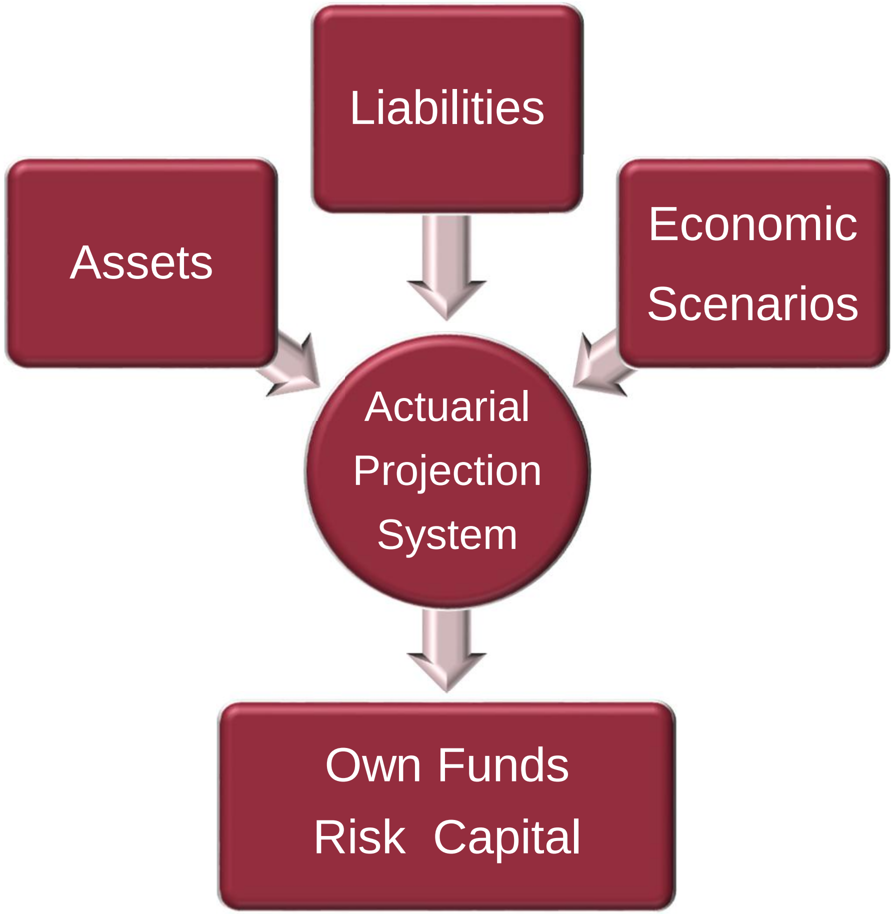
Monte-Carlo simulation is required to determine the value of financial options and guarantees embedded in life, health and pension insurance policies



Requirements on ESG

- Multi-asset (hybrid) economic model without risk premiums
- Stable projections over very long time horizons of 60-100 years
- Good simultaneous fit to liquid market data
  - *Interest rates*
  - *Interest rate volatility*
  - *Equity volatility*
  - *Inflation*
  - *FX volatility*

Economic Scenario Generator

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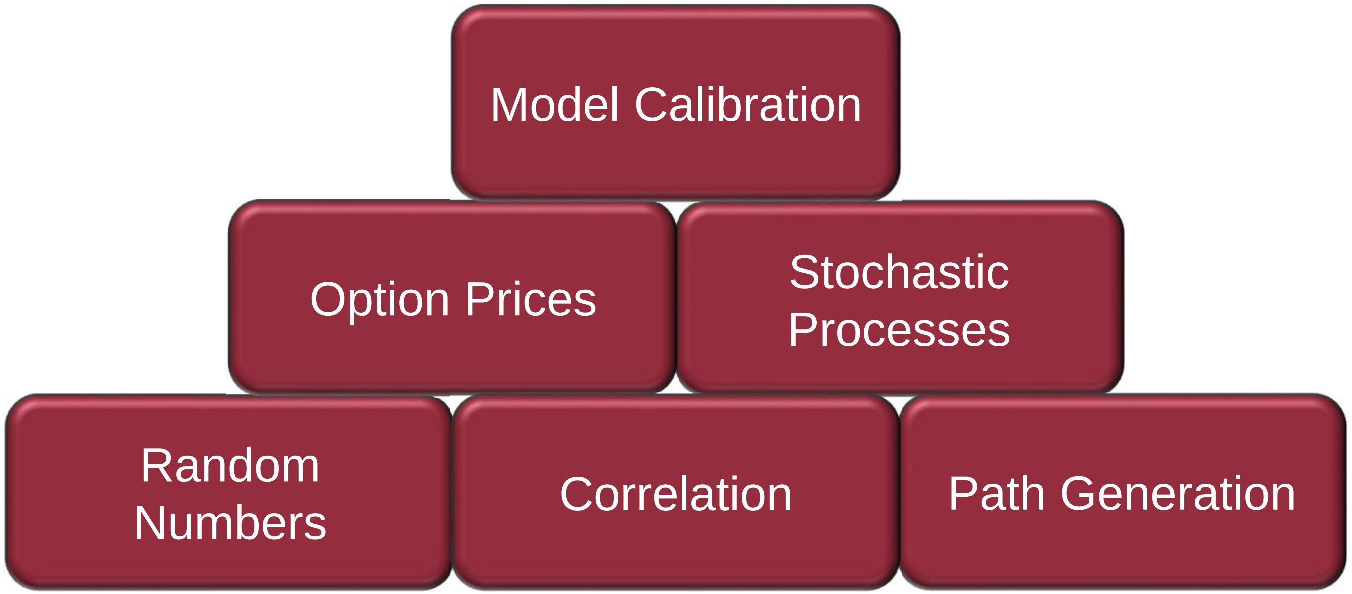
- ✓ *In-house developed in C++ / C# using QuantLib*
- ✓ *.NET library which can be used in applications supporting .NET framework*
  - *In-house developed actuarial and financial applications*
  - *VBA (e.g. in Excel) and other applications supporting .NET*
- ✓ *Configurable via .NET interface or using Excel or Access*
- ✓ *Supports calibration, analytical pricing and "on the fly" simulation of hybrid models*
  - *Interest rates:*
    - 1- and 2-Factor Hull-White
    - Cox-Ingersoll-Ross
    - Libor Market Model
  - *Equity:*
    - Black-Scholes-Merton
    - Heston
  - *Inflation*

Building Economic Scenario Generator

Idea: just put the bricks together



QuantLib offers a big variety of building blocks for financial engineering

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Uniform Random Number Generators

- Mersenne Twister: standard RNG with very long period $2^{19937}-1$
- L'Ecuyer generator
- Knuth's linear congruential generator

Gaussian Random Number Generators

- Box-Muller transformation
- Inverse cumulative Gaussian

Low Discrepancy Sequences

- Sobol
- Faure
- Halton

Correlation Matrix

- Cholesky decomposition
- Principal Component decomposition

Class **MultiPath** *contains list of correlated paths for all assets.*

MultiPath (**Size** nAsset, const **TimeGrid** &timeGrid)

Class Template **MultiPathGenerator<GSG>** *generates a* **MultiPath** *from random number generator*

Public Types

typedef **Sample**< **MultiPath** >	**sample_type**

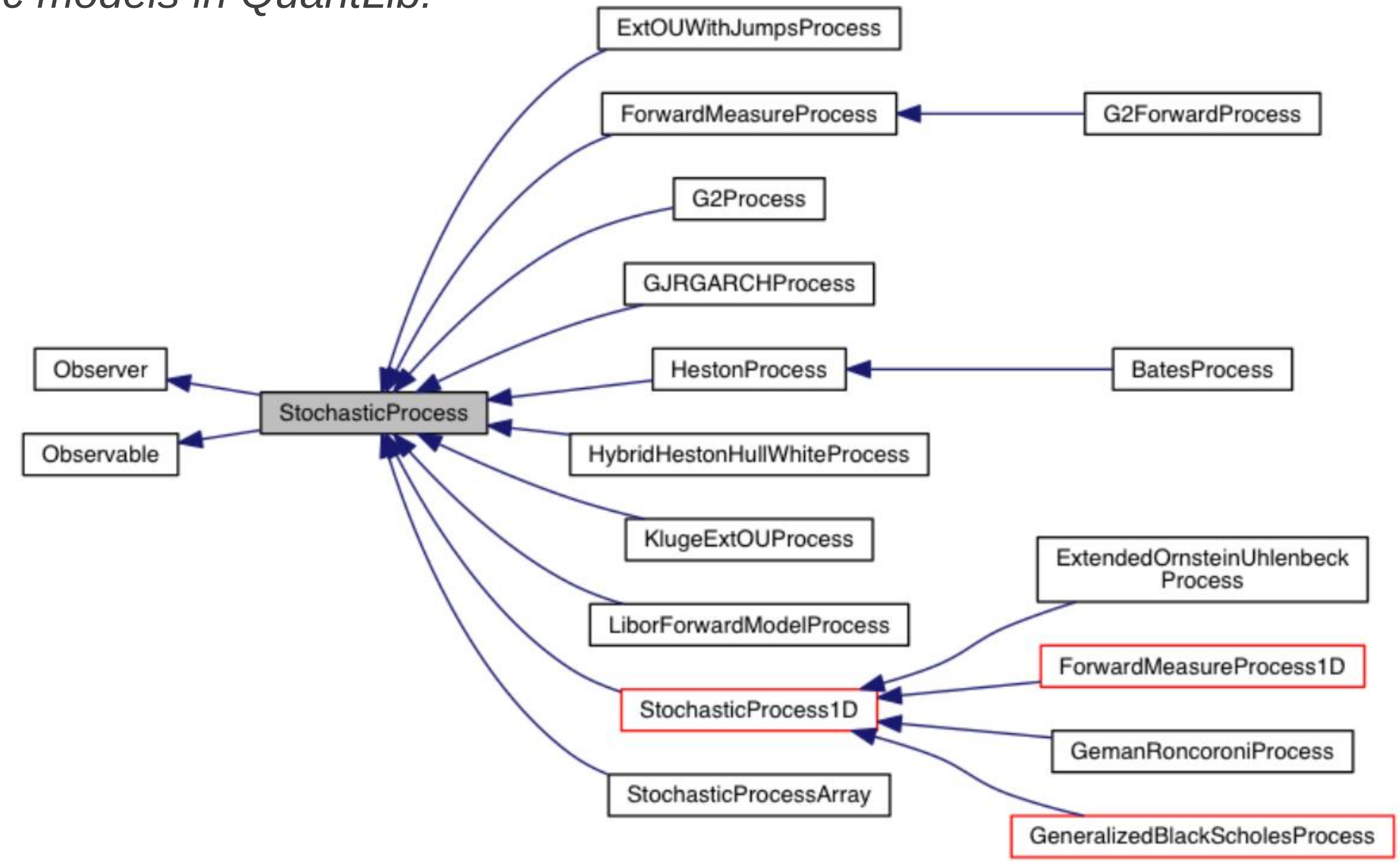
Public Member Functions

	MultiPathGenerator (const boost::shared_ptr< **StochasticProcess** > &, const **TimeGrid** &, GSG generator, bool brownianBridge=false)
const **sample_type** &	**next** () const
const **sample_type** &	**antithetic** () const

Does not support Brownian bridge (yet)!

Building Economic Scenario Generator
Monte-Carlo Framework in QuantLib

Asset dynamic is defined in a class **StochasticProcess**. *This class describes a stochastic process governed by* $dx_t = \mu(t, x_t)dt + \sigma(t, x_t)dW_t$. *It is the base class for all stochastic models in QuantLib:*



Single asset models from QuantLib need to be integrated in a consistent hybrid framework

Interest rates

- Hull-White
- Cox-Ingersoll-Ross
- G2
- Libor Market Model

Calibration to Normal or Black-76 quotes of swaptions or caplets

Equities

- Black-Scholes-Merton
- Heston
- Bates

Calibration to Black-Scholes quotes possibly with skew

FX

- Garman-Kolhagen

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Libor Market Model was the model of choice at Munich RE Group for the Solvency II preparatory phase (2006 ff.)

Forward rate dynamic:

$$\frac{df_k(t)}{f_k(t)} = \mu\left(\bar{f}, t\right)dt + \sigma_k(t)dW_k(t)$$

Advantages:

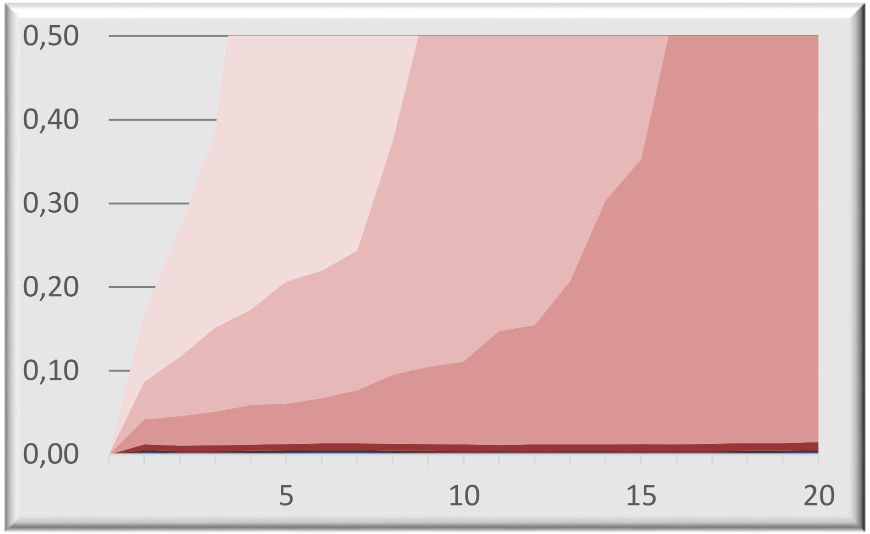
- Well known in the market
- Good fit to interest rates and ATM swaption volatilities
- Analytical approximations of swaption implied voloatilities
- Fast calibration
- No negative rates

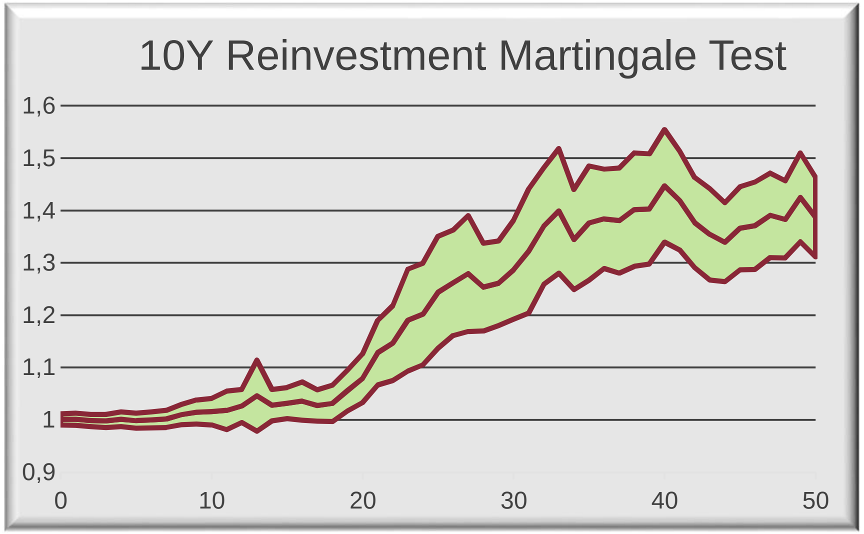
Disadvantages:

- Rates explosion
- No negative rates
- No volatility skew

Libor Market Model
Coping with Exploding Rates

- Actuarial projection systems became unstable and implausible in case of very high interest rates (>30% − 40%).
- *Naïve capping* of interest rates can produce leakage (violation of martingale property) and significantly distort NAV and risk sensitivity figures.
- *Example:* Investment in cash total return index for t years and reinvestment in 10Y zero coupon bond. This self-financing investment strategy should satisfy martingale property.





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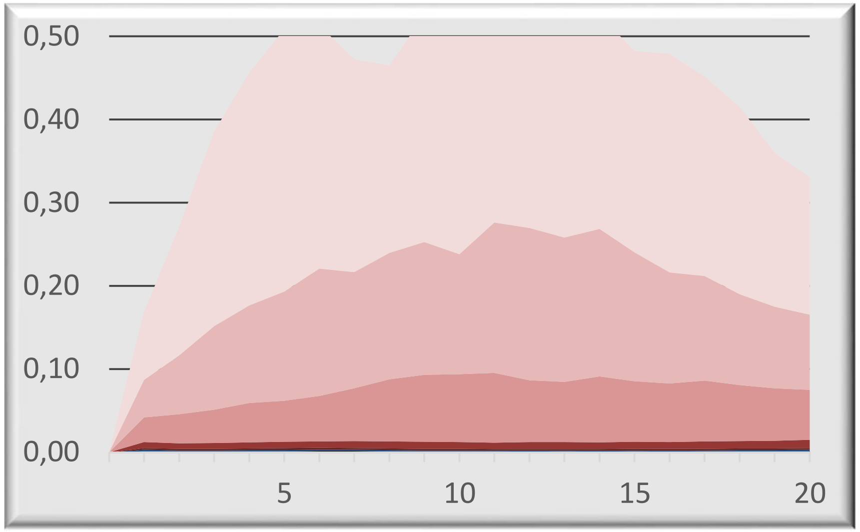
Path freezing instead of naïve capping eliminates leakage in actuarial projection models and investment strategies

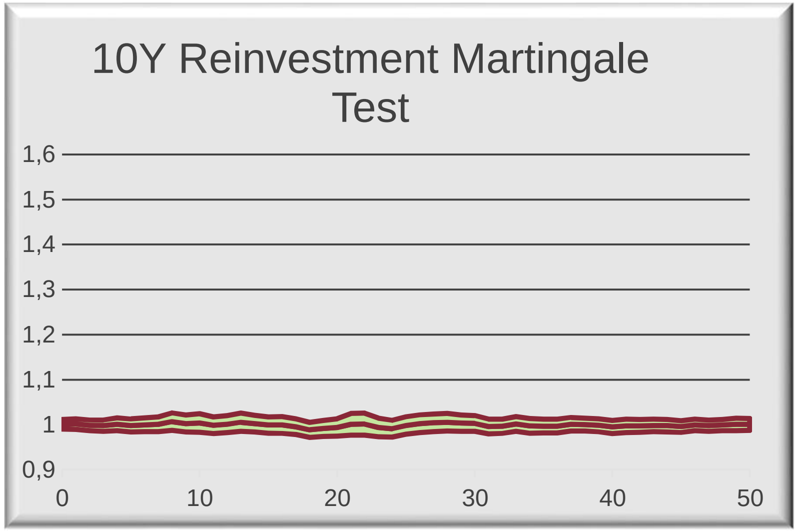
Idea:

If some forward rate exceeds the capping threshold at some time step in a given scenario, freeze the forwards dynamics from this time step (set the volatility of all forwards to zero)

Why it works:

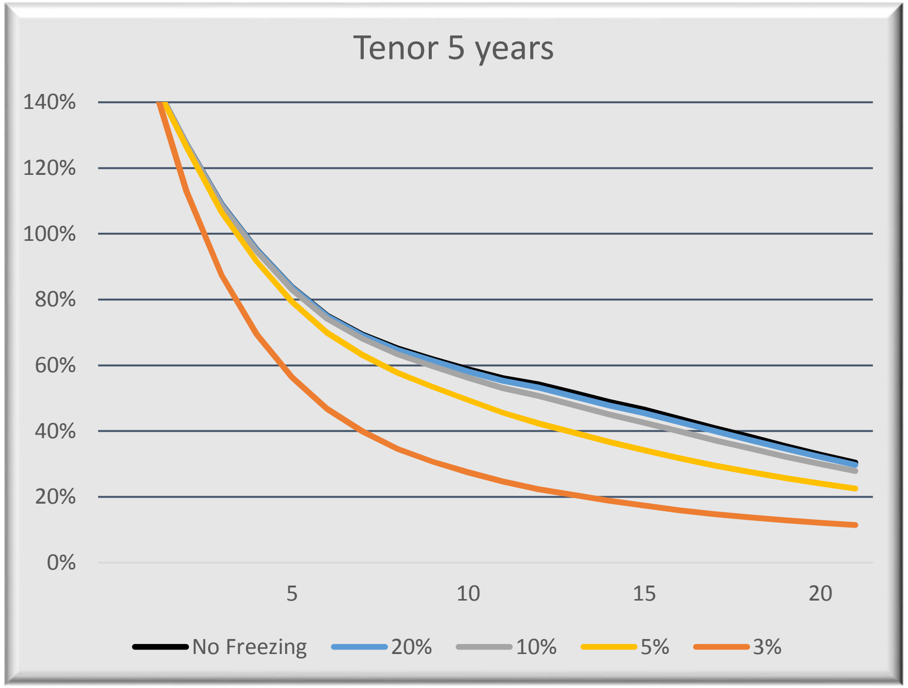
Stopped martingale is again a martingale. Freezing condition is a stopping time.

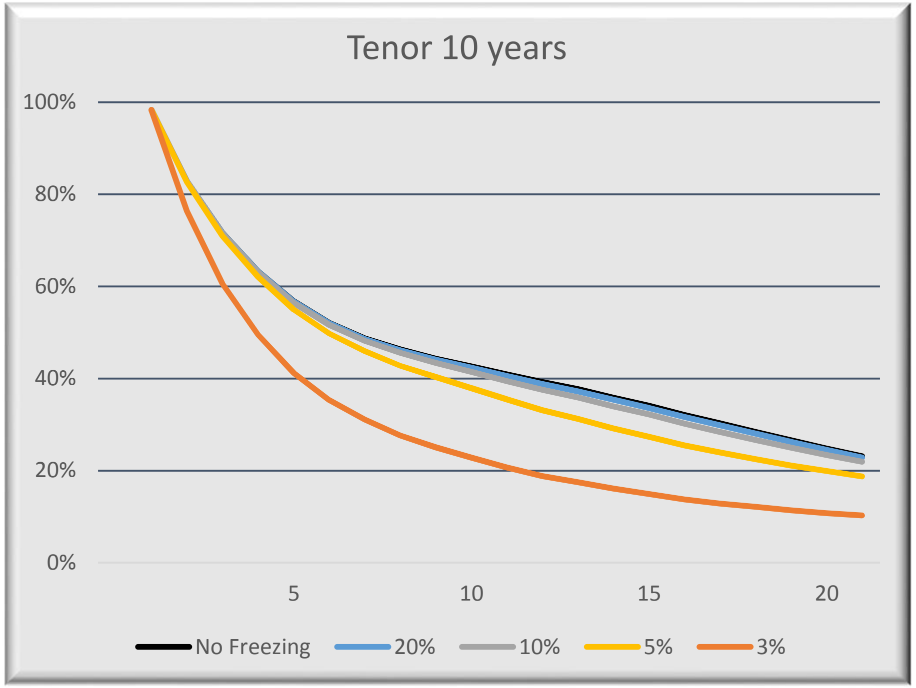




Coping with Exploding Rates
Path Freezing effect on implied volatilities

Effect of path freezing on model implied volatilities for "reasonable" freezing levels is negligible





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Existence of negative rates can not be ignored anymore.

Negative Rates

EUR Swap	< 5Y
EIONIA	< 7Y
EIOPA	< 7Y
Germany	< 10Y
CHF	< 15J

Idea:

Include displacement in forward rate dynamics

$$\frac{df_k(t)}{f_k(t)+\delta} = \mu^*\left(\bar{f},t\right)dt + \sigma_k^*(t)dW_k(t)$$

Advantage:

Existing analytics, user experience and QuantLib implementation can easily be adapted to displaced version

- *The drift term $\mu^*\left(\bar{f},t\right)$ is determined by no-arbitrage arguments.*
- *Analytical approximations for swaption implied volatilities can be derived following the arguments for non-displaced case:*
  - *Use "coefficient freezing" approximation to relate model parameters to volatilities in displaced Black-76 model*
  - *Price swaptions using analytical formulas for displaced Black-76*
  - *Transform swaption prices to volatility quotes (Black-76 or Normal)*
- *Parametrization of instantaneous volatility $\sigma_k^*(t)$ might need to be revisited for displaced case.*

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Black-76 model implied volatilities

Analytical approximation v.s. Monte-Carlo simulation

Adaptation of the “frozen coefficient” technique leads to a good analytical approximation of swaption implied volatilities

Some insurance policies are sensitive to interest rate implied volatility skew

- Libor Market Model has (almost) no skew (in Black-76 space)
- Displaced Libor Market Model is not flexible enough to reflect observed market skew

Market standard: SABR-Model

$$df_k(t) = \sigma_k(t)\, f_k(t)^{\boldsymbol{\beta}}\, dW_k(t)$$

$$d\sigma_k(t) = \boldsymbol{\alpha}\, \sigma_k(t)\, dZ_k(t)$$

$$dW_k(t)\, dZ_k(t) = \boldsymbol{\rho}\, dt$$

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Popular in insurance industry: (Displaced) Wu & Zang Model [1]

Idea: (Displaced) Libor Market Model with Heston-like stochastic volatility

$$\frac{df_k(t)}{f_k(t)+\delta} = \mu^*(\bar{f},t)dt + \sqrt{V(t)}\,\sigma_k^*(t)dW_k(t)$$

$$dV(t) = \kappa(\theta - V(t))dt + \epsilon\sqrt{V(t)}\,dZ(t)$$

$$dW_k(t)\,dZ(t) = \rho_k\,dt$$

Swaption prising:

- "Freezing" the coefficients leads to Heston-like equation for forward swap rates.
- Adapt Heston's arguments to derive the analytic expressions for moment generating functions for forward rates.
- Swaption prises can be obtained by numerical integration.

[1] Wu, L. and Zhang, F. "Libor Market Model With Stochastic Volatility", Journal of Industrial & Management Optimization, Volume 2, Number 2, May 2006, pp. 199-227

Interest Rate Volatility Skew
Implementation of Wu & Zang Model using QuantLib

- *Stochastic Process:* combine Cox-Ingersoll-Ross for stochastic volatility with displaced Libor Market Model.

- *Semi-analytical Pricing and Calibration:* reuse Heston analytics from QuantLib

Summary

- *Pricing of life, pension and health insurance liabilities within Solvency II regulatory framework requires advanced financial models*

- *QuantLib offers a big variety of ready to use components to build advanced multi asset hybrid models and deal with modelling challenges at insurance companies*

- *Open source architecture enables fast and efficient adaptation of financial models to insurance specific requirements*

- *A very time consuming part when using QuantLib for production is documentation*