A new Pricing Engine

for Arithmetic Average Price Options

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Literature

- Paul Wilmott on Quantitative Finance; 3-Volumes
- Jan Vecer; A unified pricing of Asian Options

Motivation

- Presently no continuous Asian option pricer based on PDE methods in QuantLib.
- While formulating i had to deal with theory. (PWQF, Prof. Jan Vecer)
- Present here application (benchmarks) in combination with theory
- Link: Vecer Patch of QuantLib (http://sourceforge.net/p/quantlib/patches/81)

Agenda

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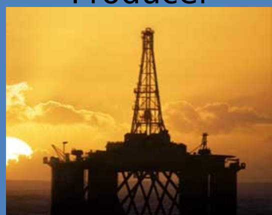
1. General Introduction: Commodity Markets and Average Options
2. Option Types and Pricing Methods
3. The classical PDE and the Vecer PDE
4. Benchmarking of the new Pricing Engine
5. A Calibration Application: Using the new Engine and Perl-SWIG to calibrate implied Black76 Volatilities from ICE Settlement Prices

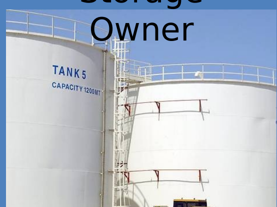
Markets

Producer



Transport

Storage Owner



FFAs

Consumer

Gasoil

Physical Market

NYMEX / ICE / Baltic Exchange

Futures / Options/ Basisswaps
(Cash / Physical Settlement)

Exchange with Clearing

- **While Vanilla Call/Put Options are the typical options for equities,**

  the average option is the classical one for commodities

- **Named Examples are:**
- TAPOs (Traded Average Price Options) on Metals at LCH
- Crude- and Fuel Oil APOs (Average Price Options) from the ICE and NYMEX/Clearport.
- Options on Freight cleared by the LCH

- **Characteristics are:**
- all of the discrete arithmetic type.
- start averaging one month before the option's expiry.
- These average options are also called Tail Asian Options (see PWQF 2nd Volume).

3.5 Fuel Oil Outright - 3.5% FOB RDAM Barges Fuel Oil Average Price Option

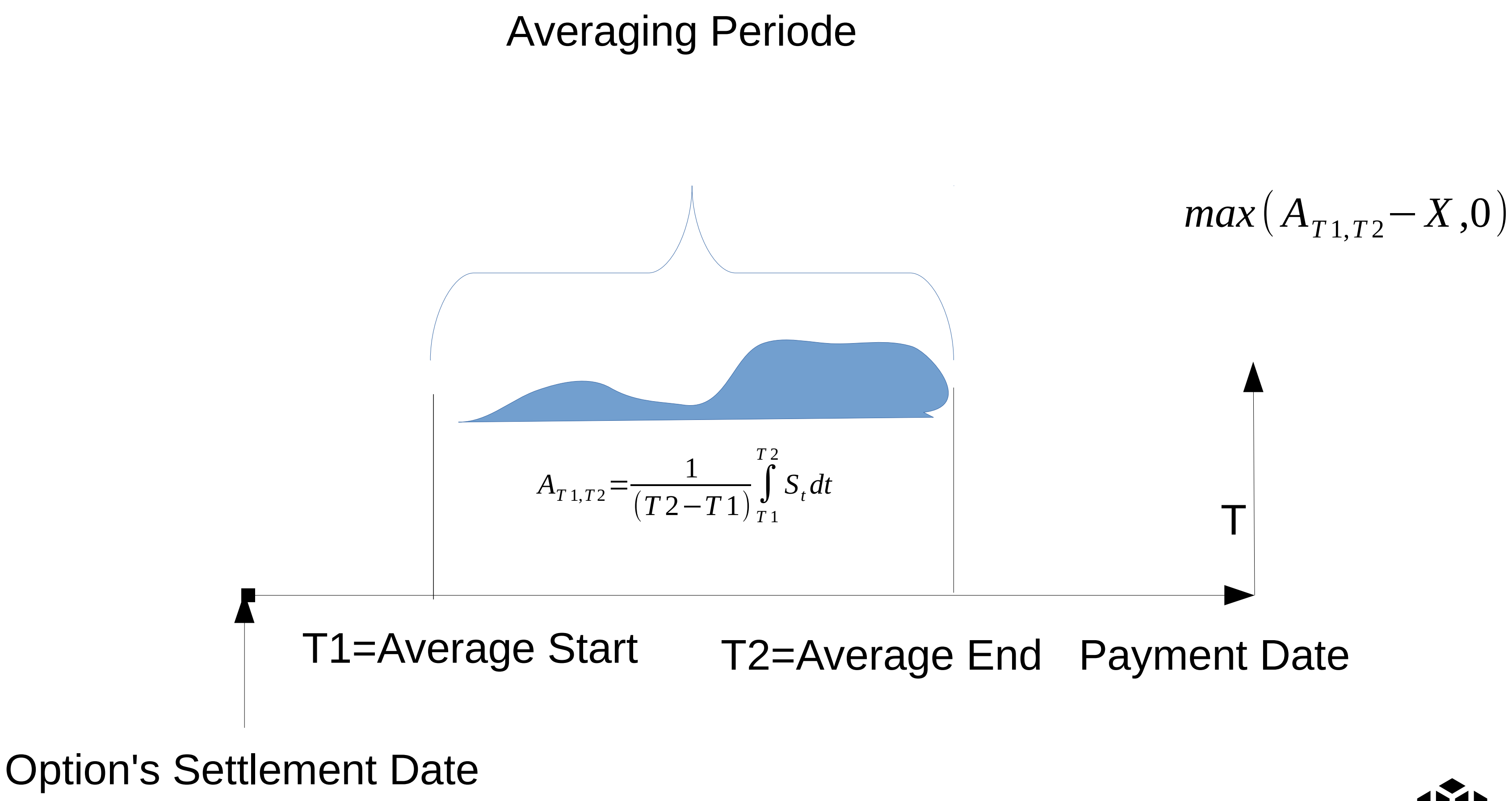
Description	The 3.5% FOB RDAM Barges Fuel Oil Average Price Option is based on the underlying 3.5% FOB RDAM Barges Fuel Oil Swap (BAR) and will automatically exercise into the settlement price of the Swap on the day of expiry of the options contract.
Hedge Instrument	The delta hedge for the 3.5% FOB Rotterdam Barges Average Price Option is the 3.5% FOB Rotterdam Barges Swap (BAR).
Contract Symbol	BAR
Contract Size	1,000 Metric Tonnes
Unit of Trading	Any multiple of 1,000 Metric Tonnes
Currency	US Dollars and cents
Trading Price Quotation	One cent ($0.01) per Metric Tonne
Settlement Price Quotation	One tenth of one cent ($0.001) per Metric Tonne
Minimum Price Fluctuation	One tenth of one cent ($0.001) per Metric Tonne
Last Trading Day	First Business Day following the settlement period
Fixed Price	The traded price or the previous day's settlement price
Floating Price	In respect of daily settlement, the Floating Price will be determined by ICE using price data from a number of sources including spot, forward and derivative markets for both physical and financial products.
Final Settlement Price	In respect of final settlement, the Floating Price will be a price in USD and cents per Metric Tonne based on the arithmetic average of the mean between the relevant high and low quotations appearing in the "Platts European MarketScan" under the heading "Northwest Europe Barges" subheading "FOB Rotterdam" for the "Fuel Oil 3.5%" quotation for each Business Day in the determination period.
Option Type	Options are Asian-style and will be automatically exercised on the expiry day if they are "in the money". The swap resulting from exercise immediately goes to cash settlement relieving market participants of the need to concern themselves with liquidation or exercise issues. If an option is "out of the money" it will expire automatically. It is not permitted to exercise the option on any other day or in any other circumstances than the Last Trading Day. No manual exercise is permitted.
Strike Price Intervals	A minimum of 10 strikes above and below at the money in $1.00 increments will be listed at launch. This contract will support Custom

A new Pricing Engine for the arithmetic average Option

Reasons for the popularity of asian options

- The Average period (asian option) as we see in the term sheet is preferred because it's more difficult to manipulate
- Average options are cheap compared to vanilla call/put options.

Payout of Average Options



Average Types

- Continuous geometric: $A_{T1,T2} = \exp\left(\frac{1}{(T2-T1)} \int_{T1}^{T2} \log(S_t)\, dt\right)$
- Discrete geometric: $A_{T1,T2} = \sqrt[n]{\prod_{i=1}^{n} S_i}$
- **Continuous arithmetic:** $A_{T1,T2} = \frac{1}{(T2-T1)} \int_{T1}^{T2} S_t\, dt$
- **Discrete arithmetic:** $A_{T1,T2} = \frac{1}{n} \sum_{i=0}^{n} S_i$

Exercise Types

- **Average Price:** $max(A_{T1,T2} - X, 0)$
- Floating Strike: $max(A_{T1,T2} - X * S_{T,0})$
- American Excercise: $max(\frac{1}{t}\int_0^t S_u du - K, 0)$ at early exercise time

State of the art Pricing Methods in QuantLib

- For asian options on the geometric average there are analytical formulas available. They are also available in QuantLib.
- For options on the arithmetic average there are no known analytical solutions available.
- For the latter numerical PDE and Monte Carlo Methods are used.
- In QuantLib the Monte Carlo and PDE methods are used for discrete arithmetic average price options

Our approach

- For the continuous arithmetic average option there are several PDE based methods available. None has yet been implemented in QuantLib.
- QuantLib uses an analytical approximation method with the Levy Engine. It is based on a lognormal assumption for the average.
- There are PDE Methods for american asians. In theory Monte Carlo could also be used.

Our approach (continued 2)

- The PDE Methods for arithmetic average options can be classified by using either one or two space dimensions.
- For Arithmetic Average Strike options PWQF gave a one (space) dimensional PDE.
- The QuantLib PDE Pricer for the discrete type uses one true space variable. But it has the average as an additional parameter The Grid thus has 2 dimensions (and time of course).
- The Vecer PDE Pricer for the continuous type presented here has one space dimension.
- The method (as presented in Vecer's paper) can be used for continuous and discrete options. It cannot be used for american asian options.

Obtaining the Pricing PDE for the continuous arithmetic average Option (we follow the reasoning in PWQF)

- Underlying Price process (real measure): $dS_t = \mu_t S_t \, dt + \sigma S_t \, dW_t$
- We introduce the running Average: $A_t = \int_0^t S_u \, du$
- Dynamic of the Running Average $dA_t = S_t \, dt$

➔ Dynamic of unhedged Derivative

$$dV(S,t,A) = \frac{\partial V(S,t,A)}{\partial t} dt + \mu S_t \frac{\partial V(S,t,A)}{\partial S} dt + S_t \frac{\partial V(S,t)}{\partial A} dt + \frac{1}{2} \sigma^2 S_t^2 \frac{\partial^2 V(S,t,A)}{\partial S^2} dt + \sigma S_t \frac{\partial V(S,t,A)}{\partial S} dW_t$$

Obtaining the Pricing PDE for the continuous arithmetic average Option (continued)

- Set up a hedging portfolio : $\Pi_t = V_t - \Delta_t S_t$
  while choosing $$\Delta_t = \frac{\partial V(S,t,A)}{\partial S}$$
- Use ITO's Lemma as before for
  the Dynamic of the hedging portfolio gives:

$$d\Pi(S,t,A) = \frac{\partial V(S,t,A)}{\partial t} dt + S_t \frac{\partial V(S,t)}{\partial A} dt + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V(S,t,A)}{\partial S^2} dt$$

- The return of the hedging Portfolio is deterministic. Thus
  the no arbitrage principal yields it should earn the risk less rate

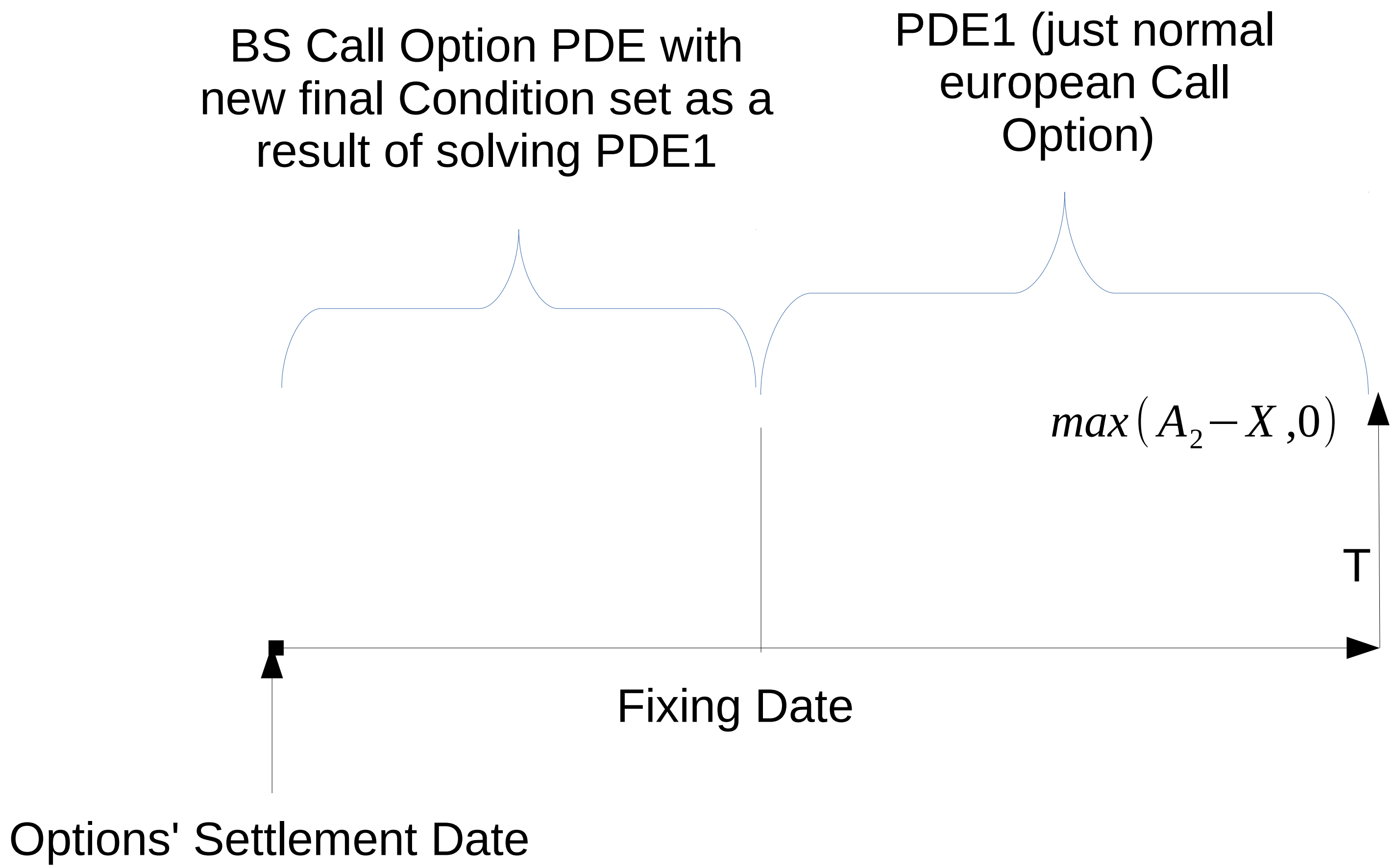
$$d\Pi_t = r\Pi dt$$

- We have a Pricing PDE:

$$rV(S,t,A) = \frac{\partial V(S,t,A)}{\partial t} + r S_t \frac{\partial V(S,t,A)}{\partial S} + S_t \frac{\partial V(S,t)}{\partial A} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V(S,t,A)}{\partial S^2}$$

with $$V(S,T,A) = max\left(\frac{1}{T}A - X, 0\right)$$

Remark: The discrete arithmetic Average Option



Obtaining the Vecer PDE by considering the Average as a traded account

- Following the Paper of Vecer we construct a self financing strategy. Because we would like to point out the similarity with the passport option problem in PWQF we use the notation used there.
- The Dynamic of the strategy looks like this: $d\pi_t = r(\pi - q(t) S_t)\,dt + q(t)\,dS_t$
- In the passport Option a trader can freely choose the strategy q(t) So at any time he can buy or sell the asset and refinance his trades by trading in the Money Market Account

Obtaining the Vecer PDE by considering the Average as a traded account (continued)

Repeating the arguments for the Pricing PDE shown before we arrive at the following PDE

$$rV(S,t,\pi)=\frac{\partial V(S,t,\pi)}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V(S,t,\pi)}{\partial S^2}+q\sigma^2 S^2\frac{\partial^2 V(S,t,\pi)}{\partial S\partial\pi}+\frac{1}{2}q^2\sigma^2 S^2\frac{\partial^2 V(S,t,\pi)}{\partial\pi^2}+rS\frac{\partial V(S,t,\pi)}{\partial S}+r\pi\frac{\partial V(S,t,\pi)}{\partial\pi}$$

With final Condition: $V(S,t,\pi)=max(\pi,0)$

This time we have to hedge with $\Delta_t=\frac{\partial V(S,t,\pi)}{\partial S}+q(t)\frac{\partial V(S,t,\pi)}{\partial\pi}$ Shares.

Obtaining the Vecer PDE by considering the Average as a traded account (continued)

- PWQF (for the passport option) and Vecer proposed the following similarity transformation:

$$V(S,t,\pi)=SH(\xi,t)$$

With :

$$\xi=\frac{\pi}{S}$$

- This gives a transformed PDE in one space dimension

$$\frac{\partial H(\xi,t)}{\partial t}+\frac{1}{2}\sigma^2(\xi-q)^2\frac{\partial^2 H(\xi,t)}{\partial \xi^2}=0$$

Final Condition in this case is: $H(\xi,T)=max(\xi,0)$

- Contrast this with the classical PDE as before

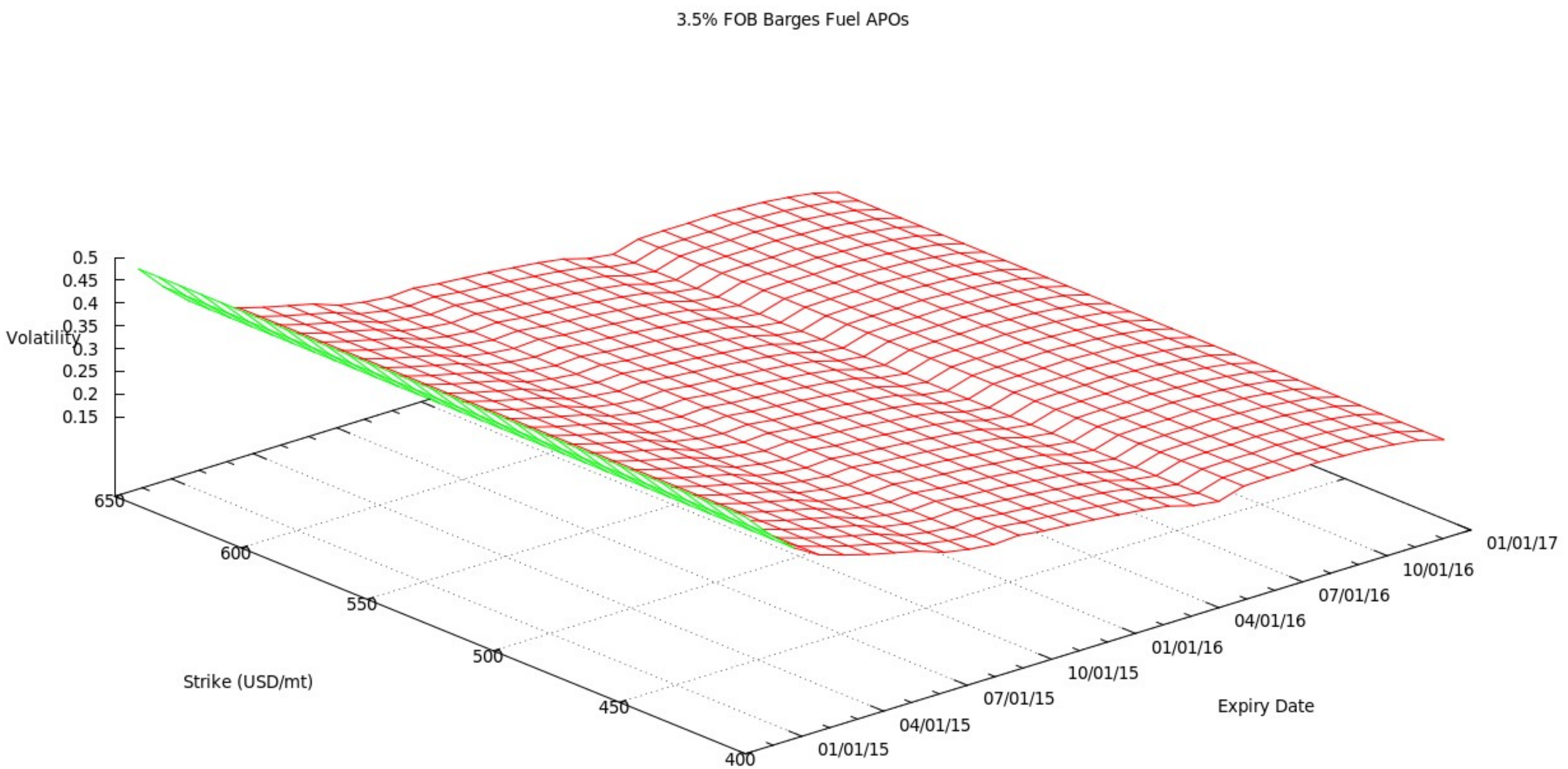
$$rV(S,t,A)=\frac{\partial V(S,t,A)}{\partial t}+rS_t\frac{\partial V(S,t,A)}{\partial S}+S_t\frac{\partial V(S,t)}{\partial A}+\frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 V(S,t,A)}{\partial S^2}$$

Benchmark with published prices

Expiry (in Years)	Spot	Rate	Volatiliy	Vecer Engine	Levy Engine	Paper	Diff Vecer (in Basis Points)	Diff Levy (in Basis Points)
1	1,9	5,00%	50,00%	0,1931730	0,1953793	0,1931740	-0,005188	11,606920
1	2	5,00%	50,00%	0,2464146	0,2497907	0,2464160	-0,007152	16,873684
1	2,1	5,00%	50,00%	0,3062202	0,3106457	0,3062200	0,001061	21,074648
1	2	2,00%	10,00%	0,0559862	0,0560537	0,0559860	0,000829	0,338613
1	2	18,00%	30,00%	0,2183857	0,2198292	0,2183880	-0,011524	7,205925
2	2	1,25%	25,00%	0,1722690	0,1734897	0,1722690	-0,000087	6,103603
2	2	5,00%	50,00%	0,3500962	0,3592044	0,3500950	0,006007	45,546776

Used 300 Asset Steps, 900 time steps
Domain was truncated at -1 and +1
Except for last Row. Here we truncated at -2 and 2

Calibration of ICE 3,5% Fuel Option Volatilities



Conclusions

- The Vecer PDE Engine gives high precision results.
- The QuantLib Pricer took less then a second for the presented benchmark results
- As presented the pricer can be used in calibration applications
- Similarity with the passport option and stochastic control makes it a promising candidate for applications like UVM and static hedging.