Pricing CMS-Spread Options with QuantLib

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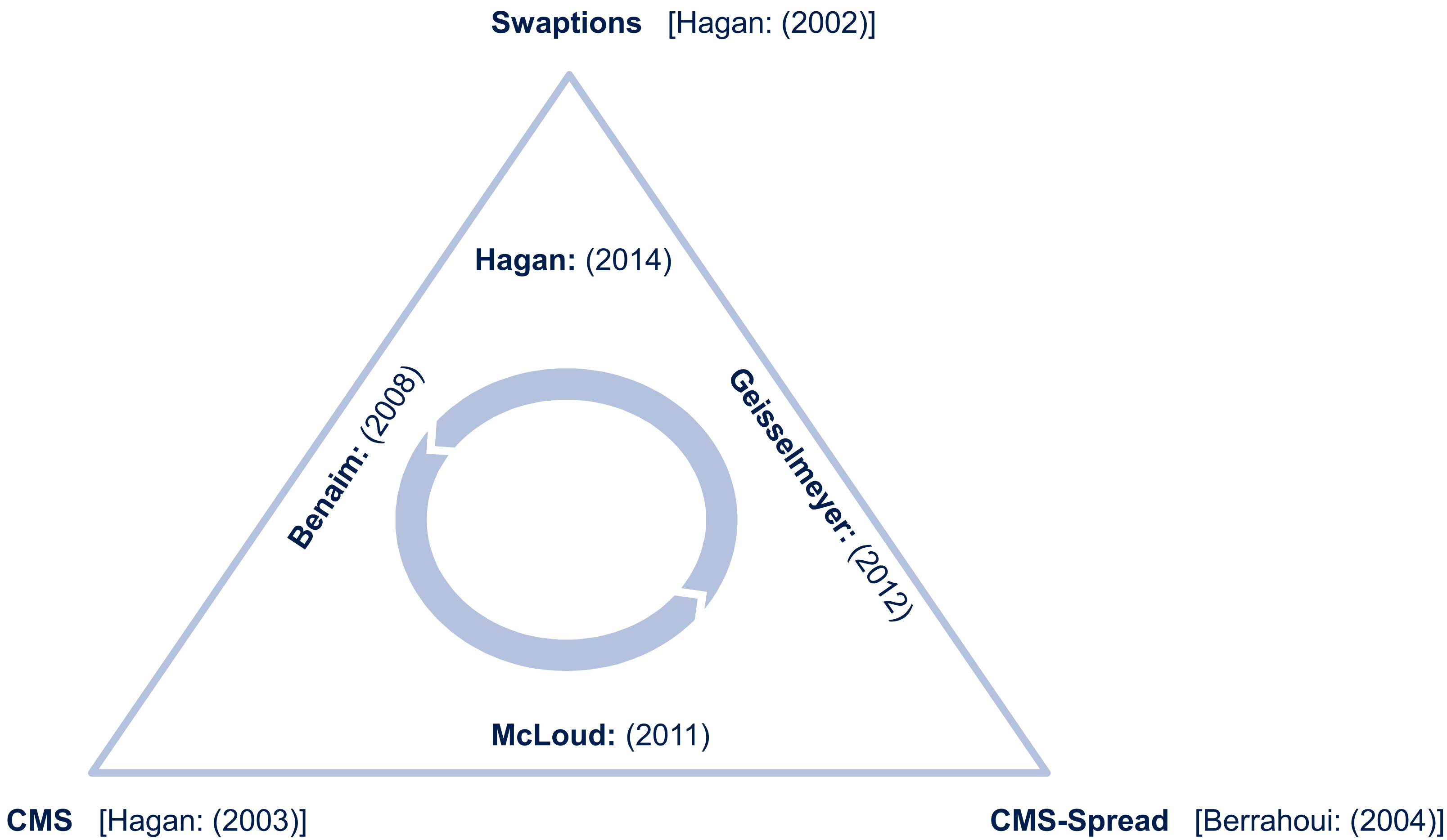
What we are going to do ?

- IR-Markets & CMS Spread Options
- Model & QuantLib-Implementation
- Calibration Results
- A Toy Model & Conclusions

This are the current challenges in the Interest Rate Markets



Arbitrage free and consistent modelling of all three edges of IR-market is still a challenging open question.

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CMS-Spread Options are always good for a surprise

» Sample Trade (CMS 10Y-2Y), T2E ≈ 14yr, K = 0.6%

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» Two Questions:

- What is a valid range for the MtM ? → e.g. from **Market Quotes**
- Whose price is the right one ? → Result of a properly **calibrated** model

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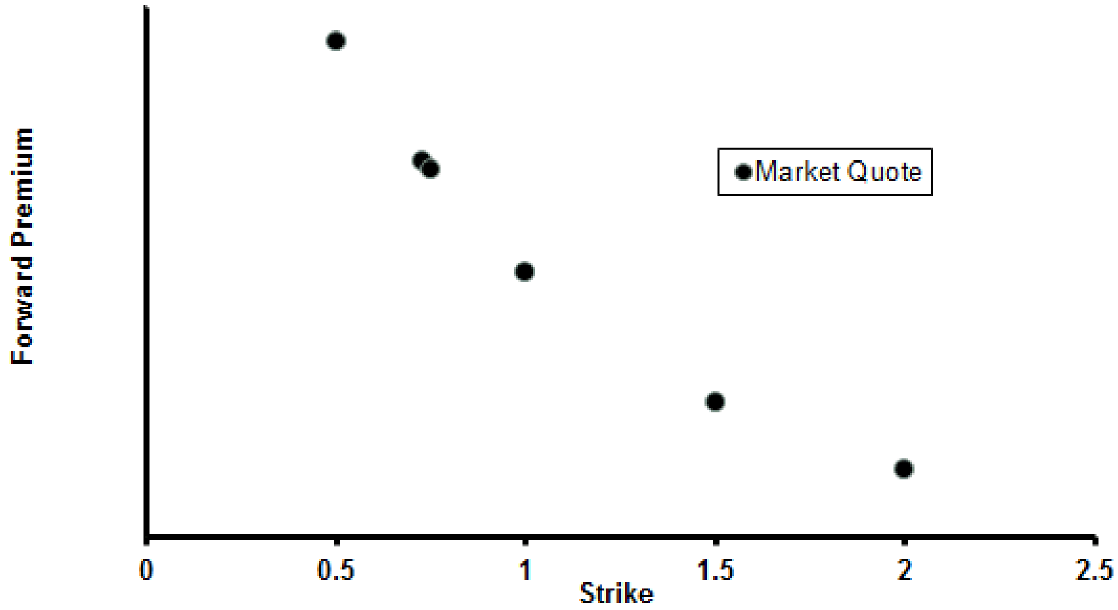
Where is the market ?

» Market Quotes of Single Look CMS-Spread Options (5Y):



Error bounds are reasonably tight. So quality of pricing model should be easily detectable.

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CMS-Spread Option Model: Forward Premium

» Berrahoui's Ansatz:

$$E[q_{10} \cdot S_{10} - q_2 \cdot S_2 - K] = \int_0^\infty \left[F_{10}\left(\frac{\tau}{q_{10}}\right) - C\left\{F_2\left(\frac{\tau + K}{q_2}\right), F_{10}\left(\frac{\tau}{q_{10}}\right)\right\} \right] \cdot d\tau$$

» **Marginals** $F_2(x)$, $F_{10}(x)$ obtained from Digital CMS-Options via a modified conundrum pricer.

$$F_i(x) = P_i(x \leq K) = 1 - \frac{Digital_i(0)}{D(T_P)}$$

» Choise of Copulas:

- (asymmetric) **Gaussian**/Gumbel

» Smile-Model:

- SABR

Marginals via CMS-Digital-Replication

$$h\,(x) = \int_{-\infty}^{\infty} \max(x-\tau)\cdot h''(\tau)\cdot d\tau$$

$$V_{digi-caplet} = A(0)\cdot E^{S}[\,\theta(S_t - K)\cdot G(S_t)]$$

$$V_{digi-caplet} = C(K)\cdot G'(K) - C'(K)\cdot G(K) + \int_{K}^{\infty} C(\tau)\cdot G(\tau)''\cdot d\tau$$

Payer-Swaption (model free)

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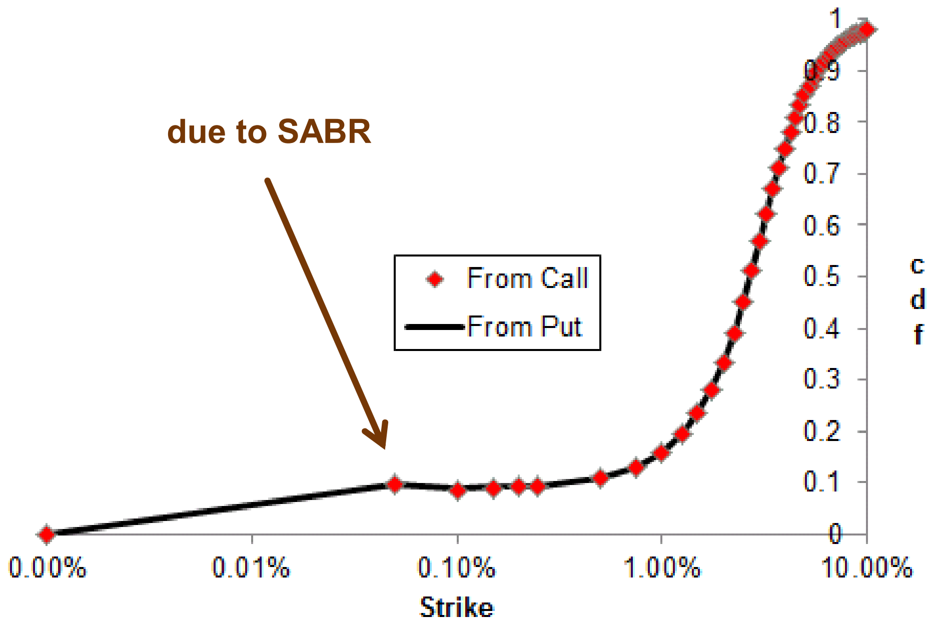
How do the Marginals look like ?

» Example: Marginal distribution of CMS2Y in 5Y:



Here a lot of work has to be done in order to cure inconsistences on the level of input market data.

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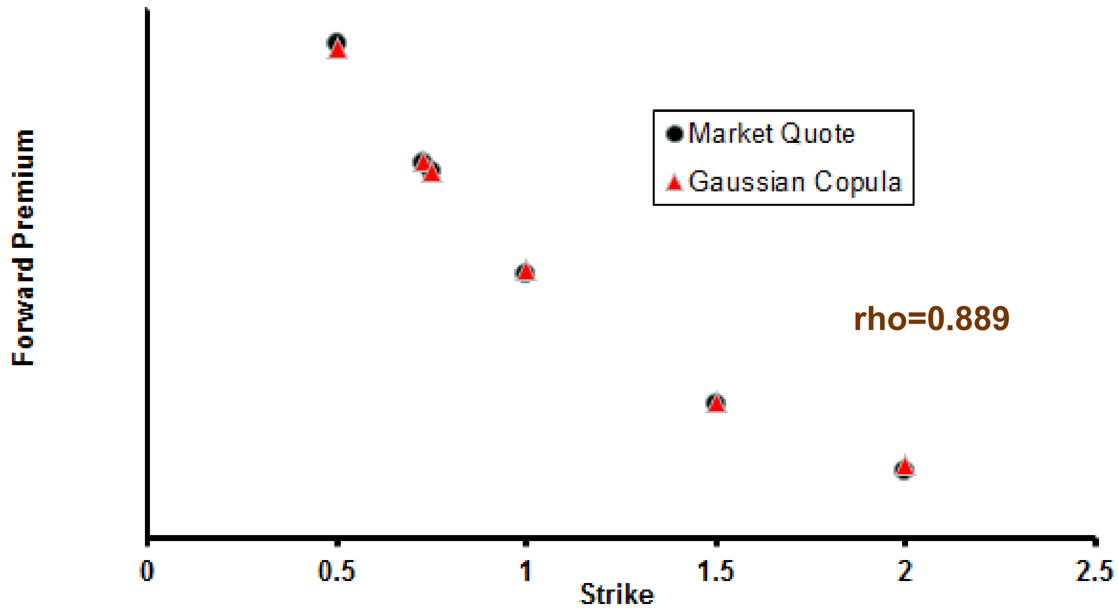
How does the implementation perform?

» Market Quotes of Single Look CMS-Spread Options:



QL results are ok. Performance is critical (several secs per coupon). Inacceptable for a productive pricer.

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Back on the road to improve stability and performance

» Steps towards increased stability:

- Alter construction of marginals, i.e. retrieve it from marginals in the swap measure:

$$F^{T_P}(x) = \int_{-\infty}^{x} \frac{G(\tau)}{G(S_0)} \cdot \rho^A(\tau) \cdot d\tau$$

- McLoud has given more symmetrical versions of the original Berrahoui formula. From a numerical perspective it might be advantageous to use this symmetric version.

Using a precomputed table of marginals one can get rid of the performance issue.

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Toy Model / VBA-Prototyping

» Toy-Modell: lognormal marginals, no swaption smile (cf. Andersen & Piterbarg, p. 778):

$$F^{T_P}(x) = \int_{-\infty}^{x} \frac{G(\tau)}{G(S_0)} \cdot \rho^A(\tau) \cdot d\tau$$

ATM-Density

Toy model is different from the standard model, that quotes CMS-Options via a normal volatility.

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Toy Model / VBA-Prototyping

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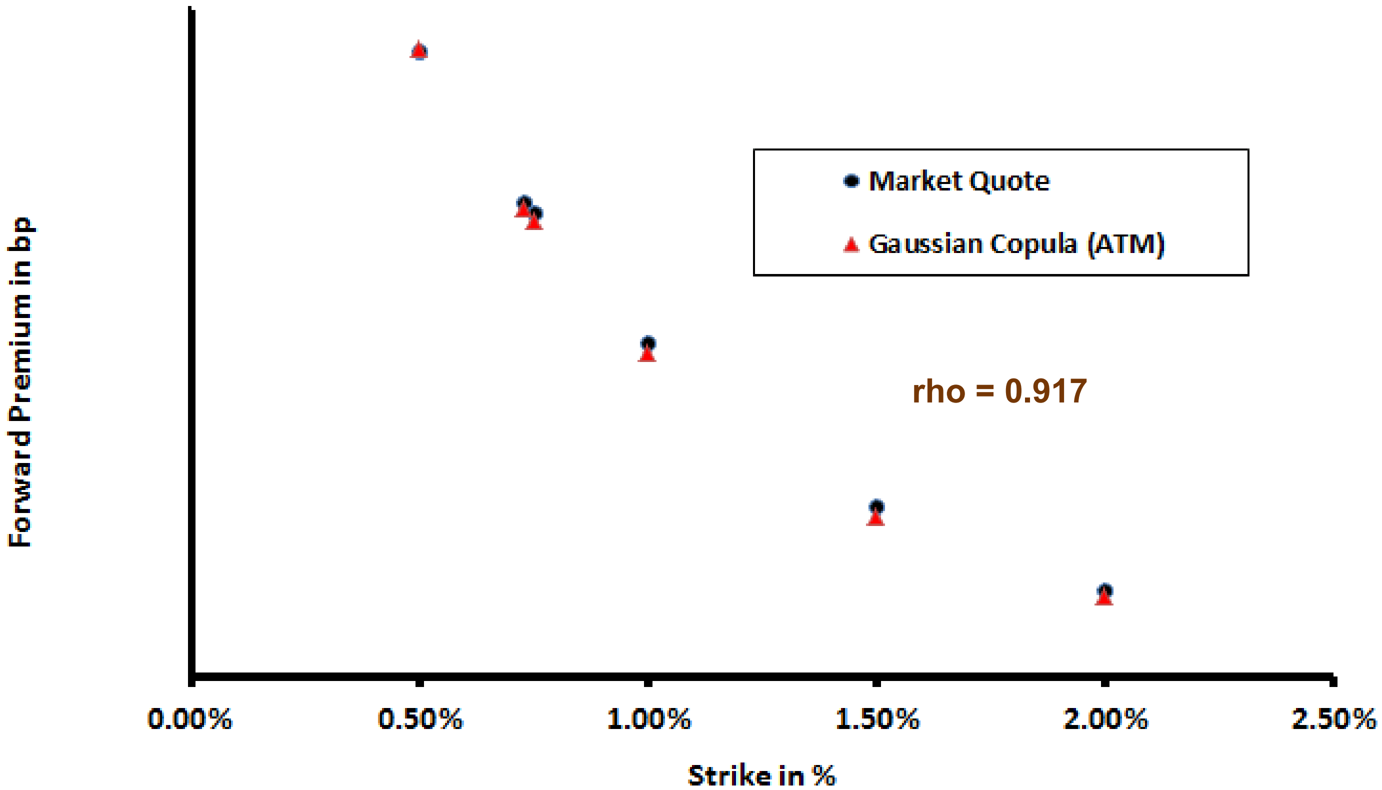

Toy model performs as good as the real implementation and is even faster. Implied parameters are similar.

Conclusions

» QL-Implementation:

- A working implementation for CMS-Spread Options is nearly directly achievable by reusing several code pieces already contained in QuantLib.
- The main task is the generation of the marginal distributions. Empirical evidence indicates that a direct construction of the marginals using the well established conundrum method is not fast enough for productive purposes.
- The prerecording of the marginal distribution before entering the integration is likely to solve this deficiency but was not implemented yet.
- Some suggestions to improve the stability of the pricer were made.
- Calibration to Market Quotes works fine.

» Toy-Model:

- In addition the study of a toy model seems to indicate, that a calibrated model can be obtained by plugging ATM marginals from the swap measure more or less directly into the copula method.
- Again, calibration to Market Quotes works fine. Parameters have slightly changed.

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References

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